

9. cvičení - výsledky

Příklad 1.

(a)

$$\int \frac{4}{2x+3} - \frac{1}{x+1} dx \stackrel{c}{=} 2 \log |2x+3| - \log |x+1|.$$

(b)

$$\int -\frac{1}{x} + \frac{1}{x+1} + \frac{1}{(x+1)^2} + \frac{2}{(x+1)^3} dx \stackrel{c}{=} \log \left| \frac{x+1}{x} \right| - \frac{1}{x+1} - \frac{1}{(x+1)^2}.$$

(c)

$$\int -\frac{2}{x-1} + \sum_{k=0}^{14} x^k dx \stackrel{c}{=} -2 \log |x-1| + \sum_{k=0}^{14} \frac{1}{k+1} x^{k+1}.$$

(d)

$$\int 1 + \frac{1}{3} \cdot \frac{1}{x^2+1} - \frac{16}{3} \cdot \frac{1}{x^2+4} dx \stackrel{c}{=} x + \frac{1}{3} \arctan x - \frac{8}{3} \arctan \frac{x}{2}.$$

(e)

$$\int 1 + \frac{1}{8} \cdot \frac{1}{x-1} - \frac{1}{8} \cdot \frac{1}{x+1} - \frac{9}{4} \cdot \frac{1}{x^2+3} dx \stackrel{c}{=} x + \frac{1}{8} \log \left| \frac{x-1}{x+1} \right| - \frac{3\sqrt{3}}{4} \arctan \frac{x}{\sqrt{3}}.$$

(f)

$$\int \frac{1}{(x-2)^2} - \frac{1}{x^2-4x+5} dx \stackrel{c}{=} -\frac{1}{x-2} - \arctan(x-2).$$

(g)

$$\int \frac{1}{x^2+2x+2} - \frac{2x+2}{(x^2+2x+2)^2} dx \stackrel{c}{=} \arctan(x+1) + \frac{1}{x^2+2x+2}.$$

(h)

$$\frac{1}{3} \int \frac{x+1}{x^2-x+1} - \frac{1}{x+1} dx \stackrel{c}{=} -\frac{1}{3} \log |x+1| + \frac{1}{6} \log(x^2-x+1) + \frac{\sqrt{3}}{3} \arctan \frac{2x-1}{\sqrt{3}}.$$

(i)

$$\int x^2 + x - 1 + \frac{2-3x}{x^2-x+2} dx \stackrel{c}{=} \frac{1}{3} x^3 + \frac{1}{2} x^2 - x - \frac{3}{2} \log(x^2-x+2) + \frac{1}{\sqrt{7}} \arctan \frac{2x-1}{\sqrt{7}}.$$

(j)

$$\frac{1}{2} \int \frac{x+1}{x^2+x+1} - \frac{x-1}{x^2-x+1} dx \stackrel{c}{=} \frac{1}{4} \log \frac{x^2+x+1}{x^2-x+1} + \frac{1}{2\sqrt{3}} \left(\arctan \frac{2x+1}{\sqrt{3}} + \arctan \frac{2x-1}{\sqrt{3}} \right).$$

(k)

$$\begin{aligned} \int -\frac{44}{9} \cdot \frac{1}{2x+1} + \frac{11}{9} \cdot \frac{2x+1}{x^2+x+1} + \frac{1}{3} \cdot \frac{7x+5}{(x^2+x+1)^2} dx \\ \stackrel{c}{=} -\frac{22}{9} \log |2x+1| + \frac{11}{9} \log(x^2+x+1) + \frac{1}{3} \cdot \frac{x-3}{x^2+x+1} + \frac{2\sqrt{3}}{9} \arctan \frac{2x+1}{\sqrt{3}}. \end{aligned}$$

(l)

$$\begin{aligned} & \frac{1}{4} \int \frac{x+1}{x^2+x+1} + \frac{-x+1}{x^2-x+1} + \frac{x+1}{(x^2+x+1)^2} + \frac{-x+1}{(x^2-x+1)^2} dx \\ & \stackrel{c}{=} \frac{1}{8} \log \frac{x^2+x+1}{x^2-x+1} - \frac{1}{6} \cdot \frac{2x^2+1}{x^4+x^2+1} + \frac{5}{12\sqrt{3}} \left(\arctan \frac{2x+1}{\sqrt{3}} + \arctan \frac{2x-1}{\sqrt{3}} \right). \end{aligned}$$

Příklad 2.

(a) $\frac{1}{2} \log |x+1| + 2 \log |x+2| - \frac{3}{2} \log |x+3|$, $x \in (-\infty, -3), (-3, -2), (-2, -1), (-1, \infty)$

(b) $\frac{1}{3} \log |x-1| - \frac{1}{6} \log(x^2+x+1) + \frac{1}{\sqrt{3}} \arctan \frac{2x+1}{\sqrt{3}}$, $x \in (-\infty, 1), (1, \infty)$

(c) $x + \frac{1}{6} \log |x| - \frac{9}{2} \log |x-2| + \frac{28}{3} \log |x-3|$, $x \in (-\infty, 0), (0, 2), (2, 3), (3, \infty)$

(d) $x + \frac{1}{3} \arctan x - \frac{8}{3} \arctan \frac{x}{2}$, $x \in \mathbb{R}$

(e) $\frac{1}{x+1} + \frac{1}{2} \log |x^2-1|$, $x \in (-\infty, -1), (-1, 1), (1, \infty)$

(f) $\log \left| \frac{x}{1+x} \right| - \frac{2}{\sqrt{3}} \arctan \frac{2x+1}{\sqrt{3}}$, $x \in (-\infty, -1), (-1, 0), (0, \infty)$

(g) $-\frac{5x-6}{x^2-3x+2} + 4 \log \left| \frac{x-1}{x-2} \right|$, $x \in (-\infty, 1), (1, 2), (2, \infty)$

(h) $\frac{1}{3} \log |x+1| - \frac{1}{6} \log(x^2-x+1) + \frac{1}{\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}}$, $x \in (-\infty, -1), (-1, \infty)$

Příklad 3.

(a) $-\frac{3}{16} \left(\frac{x}{2(1+x^2)} + \frac{1}{2} \arctan x \right)$, $x \in \mathbb{R}$

(b) $\frac{1}{3} \frac{2x+1}{1+x+x^2} + \frac{4}{3\sqrt{3}} \arctan \frac{2x+1}{\sqrt{3}}$, $x \in \mathbb{R}$

(c) $\arctan(x+1) + \frac{1}{x^2+2x+2}$, $x \in \mathbb{R}$

(d) $\frac{3}{8} \arctan x + \frac{3}{8} \frac{x}{x^2+1} + \frac{1}{4} \frac{x}{(x^2+1)^2}$, $x \in \mathbb{R}$

(e) $\frac{1}{3} \frac{5x+2}{x^2+x+1} + \frac{8}{3\sqrt{3}} \arctan \frac{2x+1}{\sqrt{3}} + \frac{1}{9} \log \frac{(x-1)^2}{x^2+x+1}$, $x \in (-\infty, 1), (1, \infty)$

(f) $\frac{5}{12\sqrt{3}} \arctan \frac{2x+1}{\sqrt{3}} + \frac{5}{12\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} + \frac{1}{8} \log \frac{x^2+x+1}{x^2-x+1} - \frac{1}{6} \frac{2x^2+1}{x^4+x^2+1}$, $x \in \mathbb{R}$

(g) $\frac{1}{9} \log \frac{(x+1)^2}{x^2-x+1} + \frac{2}{3\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} + \frac{1}{3} \frac{x}{x^2+1}$

(h) $-\frac{1}{10} \frac{x^5+2}{x^{10}+2x^5+2} - \frac{1}{10} \arctan(x^5+1)$, $x \in \mathbb{R}$

Příklad 4.

(a) $\log x + \log (\log^2 x - \log x + 1) + \frac{2}{\sqrt{3}} \arctan \frac{2 \log x - 1}{\sqrt{3}}$, $x \in (0, \infty)$

(b) potřeba přepočítat

(c) $\frac{7}{6} \log |x+3| - \frac{1}{12} \log(x^2+2x+3) - \frac{1}{3} \sqrt{2} \arctan \frac{x+1}{\sqrt{2}}$, $x \in (-\infty, -3), (-3, \infty)$

(d) $\log |x+1| + \frac{1}{11} \frac{x-5}{x^2+x+3} + \frac{24}{11\sqrt{11}} \arctan \frac{2x+1}{\sqrt{11}}$, $x \in (-\infty, -1), (-1, \infty)$

(e) $\frac{x^2}{2} - x + \frac{1}{3} \log |x+1| - \frac{2}{3} \log(x^2+x+3) + \frac{2\sqrt{11}}{3} \arctan \frac{2x+1}{\sqrt{11}}$, $x \in (-\infty, -1), (-1, \infty)$